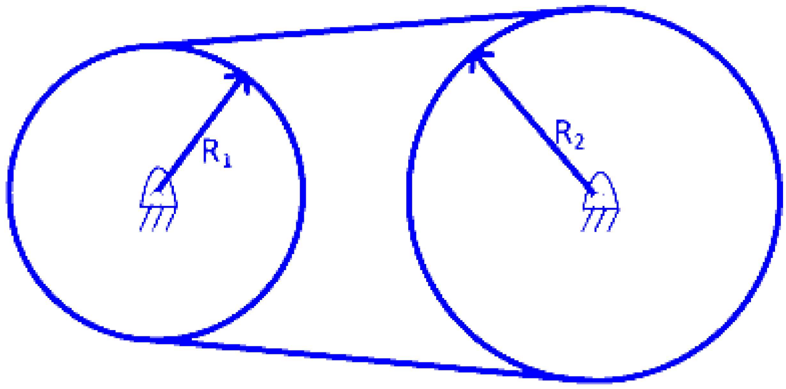
Rotational Motion

Question1

Two fly wheels are connected by a non-slipping belt as shown in the figure. $I_1 = 4 \text{ kg m}^2, r_1 = 20$ cm, $\mathrm{I}_2 = 20 \text{ kg m}^2$ and $\mathrm{r}_2 = 30$ cm. A torque of 10 Nm is applied on the smaller wheel. Then match the entries of column I with appropriate entries of column II.

I	Quantities	II	Their numerical Values (in SI units)
(a)	Angular acceleration of smaller wheel	(i)	$\frac{5}{3}$
(b)	Torque on the larger wheel	(ii)	$\frac{100}{3}$
(c)	Angular acceleration of larger wheel	(iii)	$\frac{5}{2}$



KCET 2025

Options:

A. $a - ii, b - iii, c - i$

B. $a - iii, b - i, c - ii$

C. $a - ii, b - i, c - iii$

D. $a - iii, b - ii, c - i$

Answer: D

Solution:

Explanation

The relationship between the torque, moment of inertia, and angular acceleration for each wheel can be established with the following equations:

For the smaller wheel:

$$T_1 = I_1 \alpha_1$$

where T_1 is the torque applied to the smaller wheel, $I_1 = 4\,\text{kg} \cdot \text{m}^2$ is its moment of inertia, and α_1 is the angular acceleration.

The angular acceleration ratio between the two wheels is given by:

$$\alpha_2 r_2 = \alpha_1 r_1$$

where α_2 is the angular acceleration of the larger wheel, $r_1 = 0.2\,\text{m}$, and $r_2 = 0.3\,\text{m}$.

Given that a torque of 10 Nm is applied to the smaller wheel:

$$10 = \alpha_1 \times 4$$

Solving for α_1:

$$\alpha_1 = \frac{10}{4} = \frac{5}{2}\,\text{rad/s}^2$$

Using the relation for the angular accelerations:

$$\alpha_2 \times 0.3 = \left(\frac{5}{2}\right) \times 0.2$$

$$\alpha_2 = \frac{5}{3}\,\text{rad/s}^2$$

Finally, for the larger wheel, the torque T_2 is given by:

$$T_2 = I_2 \alpha_2$$

where $I_2 = 20\,\text{kg} \cdot \text{m}^2$. Plugging in the values:

$$T_2 = 20 \times \frac{5}{3} = \frac{100}{3}\,\text{Nm}$$

Thus, the angular accelerations and the torque can be summarized as follows:

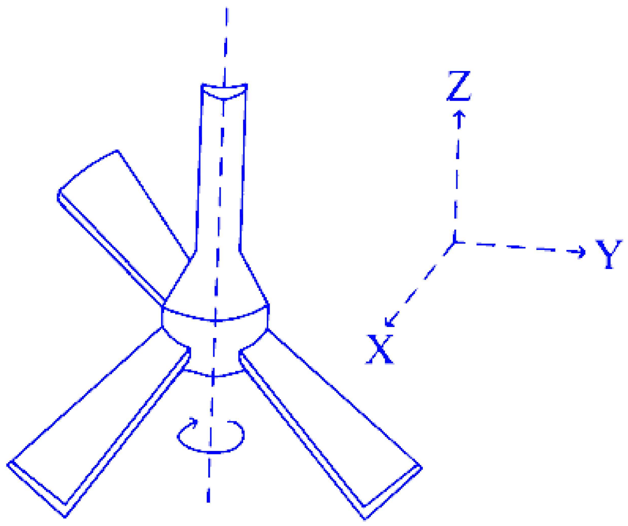
Angular acceleration of the smaller wheel $\alpha_1 = \frac{5}{2}\,\text{rad/s}^2$

Angular acceleration of the larger wheel $\alpha_2 = \frac{5}{3}\,\text{rad/s}^2$

Torque on the larger wheel $T_2 = \frac{100}{3}\,\text{Nm}$

Question2

A ceiling fan is rotating around a fixed axle as shown. The direction of angular velocity is along ....



KCET 2024

Options:

A. $+\hat{\mathbf{j}}$

B. $-\hat{\mathbf{j}}$

C. $+\hat{\mathbf{k}}$

D. $-\hat{\mathbf{k}}$

Answer: D

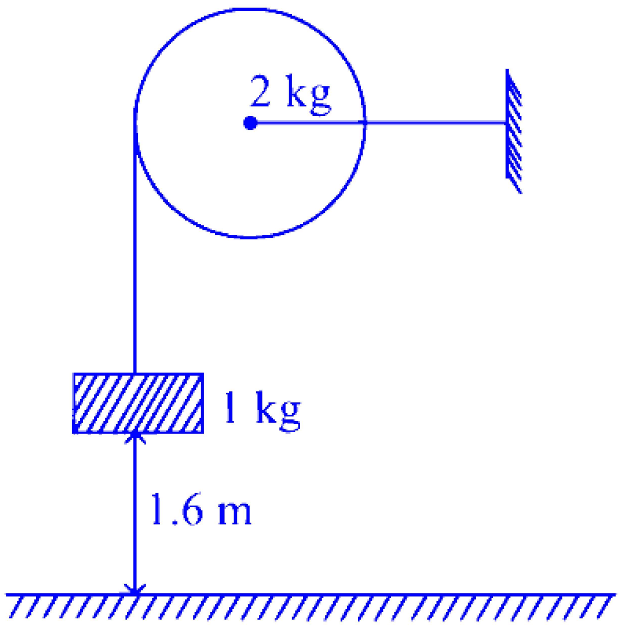
Solution:

According to right hand rule, direction of angular velocity is along Z-axis.

i.e., Along $-\hat{\mathbf{k}}$

Question3

A body of mass 1 kg is suspended by a weightless string which passes over a frictionless pulley of mass 2 kg as shown in the figure. The mass is released from a height of 1.6 m from the ground. With what velocity does it strike the ground?



KCET 2024

Options:

A. $16 \mathrm{~ms}^{-1}$

B. $8 \mathrm{~ms}^{-1}$

C. $4 \sqrt{2} \mathrm{~ms}^{-1}$

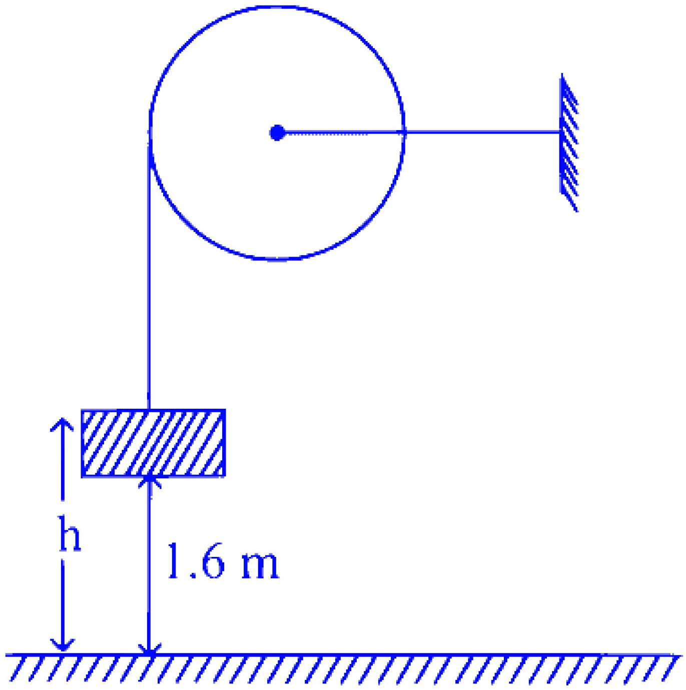
D. $4 \mathrm{~ms}^{-1}$

Answer: D

Solution:

Given, mass of the body, $m_1=1 \mathrm{~kg}$

Mass of pulley, $m_2=2 \mathrm{~kg}$



According to conservation of energy,

$$m_1gh = \frac{1}{2}m_1v^2 + \frac{1}{2}I\omega^2$$
$$= \frac{1}{2}\omega_1v^2 + \frac{1}{2} \times \frac{m_2R^2}{2} \times \left(\frac{v}{R}\right)^2$$

$\left[\because I = \frac{m_2R^2}{2} \text{ and } \omega = \frac{v}{R}\right]$

$$\Rightarrow \quad m_1gh = \frac{1}{2}m_1v^2 + \frac{1}{4} \times m_2v^2$$
$$\Rightarrow 1 \times 10 \times 1.6 = \frac{1}{2} \times 1 \times v^2 + \frac{1}{4} \times 2 \times v^2$$
$$\Rightarrow \quad v^2 = 4 \text{ m/s}$$

Question4

The moment of inertia of a rigid body about an axis

KCET 2023

Options:

A. does not depend on its mass.

B. does not depend on its shape.

C. depends on the position of axis of rotation.

D. does not depend on its size.

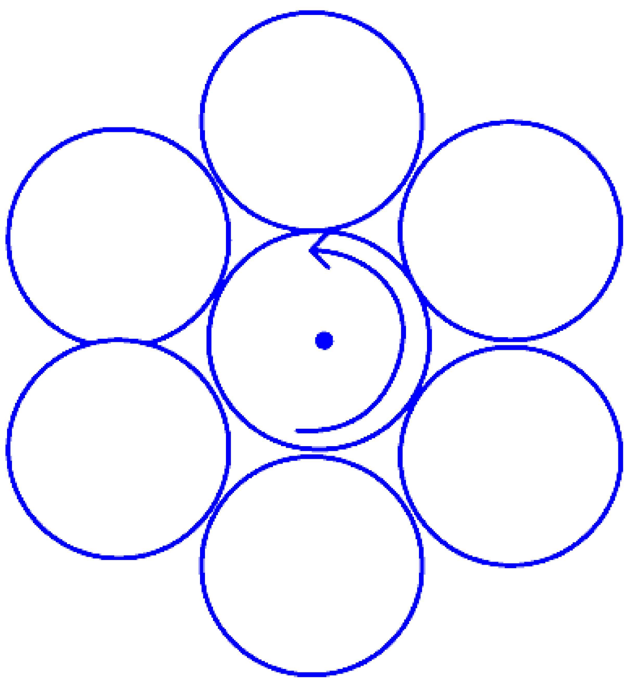
Answer: C

Solution:

Moment of inertia of a body depends on the mass of the body, distribution of mass in the body, position of axis of rotation of the body and on the distance from the axis of rotation.

Question5

Seven identical discs are arranged in a planar pattern, so as to touch each other as shown in the figure. Each disc has mass m radius R. What is the moment of inertia of system of six discs about an axis passing through the centre of central disc and normal to plane of all discs?



KCET 2023

Options:

A. $27\ mR^2$

B. $100\ mR^2$

C. $55\frac{mR^2}{2}$

D. $85\frac{mR^2}{2}$

Answer: C

Solution:

According to question, moment of inertia of 7 discs will be

$$I = I_{\text{COM}} + Md_2$$
$$= \frac{7}{2}MR^2 + 6M(2R)^2$$

($\because$ radius of all 6 discs will be $2R$ from centre).

$= \frac{55}{2}MR^2$

Question6

The angular speed of a motor wheel is increased from 1200 rpm to 3120 rpm in 16 s. The angular acceleration of the motor wheel is

KCET 2022

Options:

A. $4\pi\ \text{rad/s}^2$

B. $6\pi\ \text{rad/s}^2$

C. $8\pi\ \text{rad/s}^2$

D. $2\pi\ \text{rad/s}^2$

Answer: A

Solution:

Given, initial angular frequency of wheel,

$f_0 = 1200\text{rpm} = \frac{1200}{60}\text{rps} = 20 \text{ rps}$

$\therefore$ Initial angular velocity,

$\omega_0 = 2\pi f_0 = 2\pi \times 20 = 40\pi\text{rad/s}$

Similarly, final angular speed,

$\omega = 2\pi \times \left(\frac{3120}{60}\right)\text{rad/s} = 104\pi\text{rad/s}$

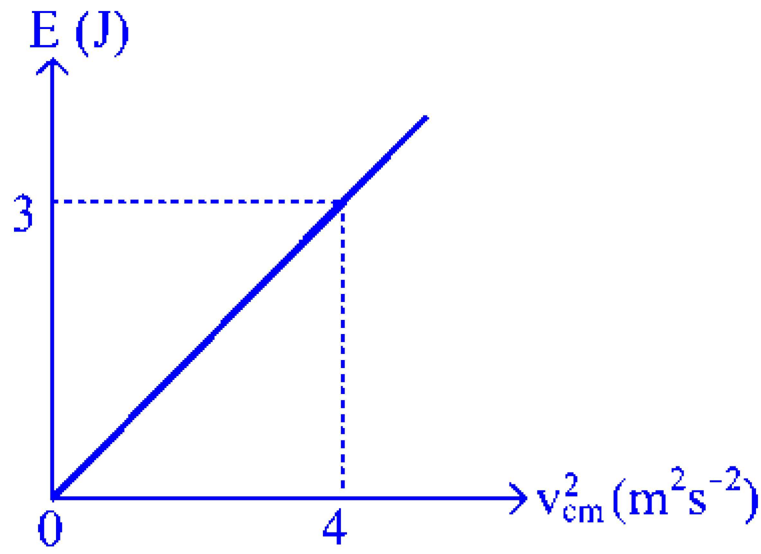
Time, $\quad t = 16 \text{ s}$

If α be the angular acceleration of the wheel, then

$$\begin{aligned} &\omega = \omega_0 + \alpha t \\ \Rightarrow \quad &\alpha = \frac{\omega - \omega_0}{t} = \frac{104\pi - 40\pi}{16} \\ &\quad = \frac{64\pi}{16} = 4\pi \text{ rad/s}^2 \end{aligned}$$

Question7

In figure E and v_{cm} represent the total energy and speed of centre of mass of an object of mass 1 kg in pure rolling. The object is



KCET 2021

Options:

A. sphere

B. ring

C. disc

D. hollow cylinder

Answer: C

Solution:

As we know, kinetic energy of an object in pure rolling motion is given as

$E = \frac{1}{2} m v_{\mathrm{cm}}^2 \left(1 + \frac{k^2}{R^2}\right)$

where, k is radius of gyration and m is the mass of the object.

$\Rightarrow \quad \frac{E}{v_{\mathrm{cm}}^2} = \frac{1}{2}\left[1 + \frac{k^2}{R^2}\right]$ .... (i)

[$\because$ Given, $m = 1$ kg]

From the given graph, substituting the value of $\frac{E}{v_{\mathrm{cin}}^2}$ in Eq. (i), we get

$$\frac{3}{4} = \frac{1}{2}\left[1 + \frac{k^2}{R^2}\right]$$
$$\frac{3}{2} - 1 = \frac{k^2}{R^2}$$
$$\frac{k^2}{R^2} = \frac{1}{2}$$

Since, for a

(a) Sphere, $\frac{k^2}{R^2} = \frac{2}{5}$

(b) Ring, $\frac{k^2}{R^2} = 1$

(c) Hollow cylinder, $\frac{k^2}{R^2} = 1$

(d) Disc, $\frac{k^2}{R^2} = \frac{1}{2}$

So, the given object is disc.

Question8

A wheel starting from rest gains an angular velocity of $10 \mathrm{rad}/\mathrm{s}$ after uniformly accelerated for 5 s. The total angle through which it has turned is

KCET 2020

Options:

A. 25 rad

B. 100 rad

C. 25π rad

D. 50π rad and a vertical axis

Answer: A

Solution:

Initial angular velocity of wheel, $\omega_0 = 0$

Final angular velocity, $\omega = 10 \mathrm{rad}/\mathrm{s}$

$t = 5$ s

By first equation of rotational motion,

$$\begin{aligned} & \omega = \omega_0 + \alpha t \\ \Rightarrow \quad & 10 = 0 + \alpha \times 5 \\ \Rightarrow \quad & \alpha = \frac{10}{5} = 2 \mathrm{rad}/\mathrm{s}^2 \end{aligned}$$

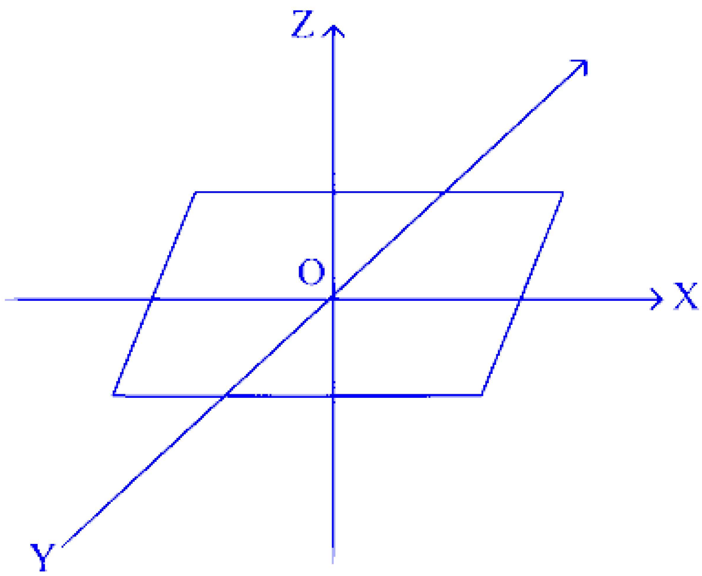
If θ be the total angle through which wheel has turned, then from,

$$\begin{aligned} & \theta = \omega_0 t + \frac{1}{2}\alpha t^2 \\ & \quad = 0 \times 5 + \frac{1}{2} \times 2 \times 5^2 = 0 + 25 \\ \Rightarrow \quad & \theta = 25 \mathrm{rad} \end{aligned}$$

Question9

A thin uniform rectangular plate of mass 2 kg is placed in xy-plane as shown in figure. The moment of inertial about x-axis is

$I_x = 0.2\ \text{kgm}^2$ and the moment of inertia about Y-axis is $I_y = 0.3\ \text{kgm}^2$. The radius of gyration of the plate about the axis passing through O and perpendicular to the plane of the plate is



KCET 2020

Options:

A. 50 cm

B. 5 cm

C. 38.7 cm

D. 31.6 cm

Answer: A

Solution:

Given, mass, $M = 2$ kg

$I_x = 0.2\ \text{kg} - \text{m}^2$
$I_y = 0.3\ \text{kg} - \text{m}^2$

and $I_y = 0.3\ \text{kg} - \text{m}^2$

According to the perpendicular axis theorem, the moment of inertia of the rectangular plate about an axis passing through O and perpendicular to the plane of plate is

$$I = I_x + I_y = 0.2 + 0.3$$
$$= 0.5 \text{ kg m}^2$$

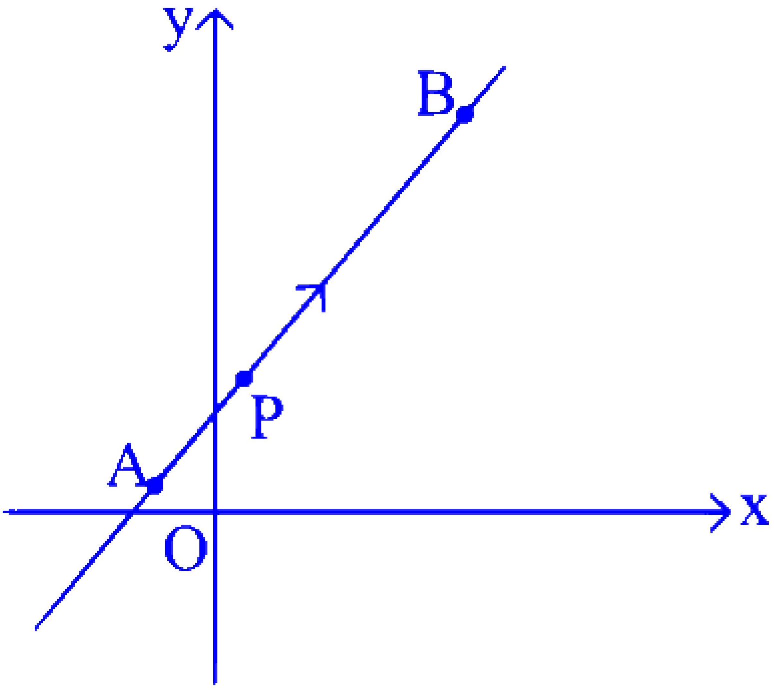
Also, $I = Mk^2$

where, $k =$ radius of gyration.

$$\Rightarrow \quad k = \sqrt{\frac{I}{M}} = \sqrt{\frac{0.5}{2}} = \sqrt{0.25} = 0.5 \text{ m or } 50 \text{ cm}$$

Question10

A particle is moving uniformly along a straight line as shown in the figure. During the motion of the particle from A to B, the angular momentum of the particle about O



KCET 2019

Options:

A. increase

B. decrease

C. remains constant

D. first increase then decrease

Answer: C

Solution:

The angular momentum of a particle moving uniformly along a straight line about a point O can be analyzed through the concept of cross product $\mathbf{L} = \mathbf{r} \times \mathbf{p}$, where $\mathbf{L}$ is the angular momentum, $\mathbf{r}$ is the position vector from the point O to the particle, and $\mathbf{p}$ is the linear momentum of the particle.

Given the uniform motion along the line from A to B:

Since the direction and magnitude of the motion remain constant,

As the particle moves from A to B, its distance from point O changes.

If the distance between O and the line of motion is constant, then the perpendicular distance from O to the particle's line of motion remains the same, keeping the angular momentum constant.

Therefore, the angular momentum of the particle about point O during its motion from A to B remains constant.

Option C: remains constant

Question11

Moment of inertia of a body about two perpendicular axes X and Y in the plane of lamina are $20 \mathrm{~kg}-\mathrm{m}^2$ and $25 \mathrm{~kg}-\mathrm{m}^2$, respectively. Its moment of inertia about an axis perpendicular to the plane of the lamina and passing through the point of intersection of X and Y-axes is

KCET 2018

Options:

A. $5 \mathrm{~kg}-\mathrm{m}^2$

B. $45 \mathrm{~kg}-\mathrm{m}^2$

C. $12.5 \mathrm{~kg}-\mathrm{m}^2$

D. $500 \mathrm{~kg}-\mathrm{m}^2$

Answer: B

Solution:

To solve this problem, we use the perpendicular axis theorem. This theorem is applicable to flat, planar objects (laminae) and states that the moment of inertia about an axis perpendicular to the plane (say the z-axis) is the sum of the moments of inertia about two mutually perpendicular axes (x and y) lying in the plane. Here's how to apply this theorem:

According to the perpendicular axis theorem:

$$I_z = I_x + I_y$$

Substitute the given values:

$$I_z = 20\,\text{kg-m}^2 + 25\,\text{kg-m}^2 = 45\,\text{kg-m}^2$$

Thus, the moment of inertia about the axis perpendicular to the plane is $45\,\text{kg-m}^2$.

The correct answer is Option B.
